Aggregate Investment Externalities and Macroprudential Regulation*

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Abstract

Evidence shows that banks tend to lend much during booms, and little during recessions. We propose a simple explanation for this phenomenon and why credit markets are dysfunctional. We show that, instead of dampening productivity shocks, the banking sector tends to exacerbate them, leading to excessive fluctuations of credit, output and asset prices. Our explanation relies on three ingredients that are characteristic of modern banks’ activities. The first ingredient is moral hazard: banks are supposed to monitor the small and medium sized enterprises that borrow from them, but they may shirk on their monitoring activities, unless they are given sufficient informational rents. These rents limit the amount that investors are ready to lend them, to a multiple of the banks’ own capital. The second ingredient is the banks’ high exposure to aggregate shocks: banks’ assets have positively correlated returns. Finally the third ingredient is the ease with which modern banks can reallocate capital between different lines of business. At the competitive equilibrium, banks offer privately optimal contracts to their investors but these contracts are not socially optimal: banks’ decisions of reallocating capital react too strongly to aggregate shocks. This is because banks do not internalize the impact of their decisions on asset prices. This generates excessive fluctuations of credit, output and asset prices. We examine the efficacy of several possible policy responses to this dysfunctionality of credit markets, and show that it can provide a rationale for macroprudential regulation.

Keywords: Bank Credit Fluctuations, Macro-prudential Regulation, Investment Externalities.

JEL: G21, G28, D86

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1 Introduction

There is now a large consensus among economists that prudential regulation of banks should also be envisaged from a systemic, or global perspective, and not only from a microeconomic point of view. The notion of macroprudential regulation, that was coined at the Bank for International Settlements (BIS) in the late 1970s, and repeatedly put forward by Borio (2003, 2010), has now become a buzzword in banking economics. However, it remains quite imprecise, since it does not rely on a universally accepted conceptual framework. Even if one restricts attention to academic publications, the motivations for macroprudential regulation are still broad and somewhat vague1.

A first strand of the literature, that includes Lorenzoni (2008), Jeanne and Korinek (2011), Korinek (2009) and Bianchi (2011) builds upon the financial accelerator mechanism identified by Bernanke and Gertler (1997) and Kiyotaki and Moore (1997). In that framework, firms and households tend to borrow too much, because they do not internalize the impact of their borrowing decisions on asset prices, and more specifically on downward spirals that occur during recessions. When borrowers make losses, they may indeed become credit constrained and be forced to sell assets, provoking a decrease in asset prices. This in turn reinforces credit constraints, leading to further asset sales and downward price spirals. This is the well-known debt-deflation mechanism identified by Fisher (1933) in his study of the Great Depression. The objective of macroprudential regulation is then to curb excessive borrowing so as to decrease the frequency and cost of banking crises. Other analyses such as Diamond and Rajan (2010) or Hansen, Kashyap and Stein (2010) rely on similar mechanisms such as the tendency of banks to issue too many short term deposits, in order to satisfy the demand of investors for (quasi-)safe assets.

Another strand of the literature relies on network externalities (see Allen, Babus and Carletti, 2010) or herding behavior of banks (see Acharya, 2009) to explain why banks should be regulated from a systemic viewpoint and not only on an individual, institution by institution, basis.

We follow here a different (but complementary) route, focusing on the notion of credit cycles. Indeed, empirical evidence shows that banks tend to lend much during booms, and little during

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1One line of argument suggests that banking regulation should have a macroeconomic component, e.g. by making bank equity requirements dependent on macroeconomic variables such as GDP growth (Repullo et al. (2009), Brunnermeier et al. (2009)), indicating that countercyclical bank equity buffers could dampen output volatility.
recessions. We propose a simple explanation for this phenomenon and show that credit markets are dysfunctional. We argue that, instead of dampening productivity shocks, the banking sector tends to exacerbate them, leading to excessive fluctuations of credit, output and asset prices. Our explanation relies on three simple ingredients that are characteristic of modern banks’ activities.

The first ingredient is moral hazard: banks are supposed to monitor the small and medium sized enterprises that borrow from them, but they may shirk on their monitoring activities, unless they are given sufficient informational rents. These rents limit the amount that investors are ready to lend them, to a multiple of the banks’ own capital. The second ingredient is the banks’ high exposure to aggregate shocks: banks’ assets have positively correlated returns. Finally the third ingredient is the ease with which modern banks can reallocate capital between different lines of business.

At the competitive equilibrium of the financial sector, banks offer privately optimal contracts to their investors but these contracts are not socially optimal: banks’ decisions of reallocating capital react too strongly to aggregate shocks. This is because banks do not internalize the impact of their decisions on asset prices. This generates excessive fluctuations of credit, output and asset prices.

We examine the efficacy of several possible policy responses to this dysfunctionality of credit markets, and show that it can provide a rationale for macroprudential regulation.

The rest of the paper is organized as follows. Section 2 discusses how our approach relates to previous literature. Section 3 presents our model. Section 4 presents two simple benchmarks (the frictionless economy and the rigid economy) in which no public intervention is warranted: in both cases competitive equilibrium leads to an efficient allocation of resources. Section 5 characterizes optimal contracts for financing banks and shows that they imply recourse to short term financing. Section 6 characterizes the competitive equilibrium. Section 7 shows that this competitive equilibrium is constrained inefficient and justifies some form of macroprudential regulation. Section 8 concludes.
2 Relation to the literature

Our work is related to several strands of the literature.

2.1 Sectoral Shocks

The role of sectoral shocks in macroeconomic fluctuations has been an important theme in the literature over the last three decades (Long and Plosser (1983), Horvath (1998, 2000), Dupor (1999), Conley and Dupor (2003), Carvalho (2009), Acemoglu et al. (2010) and Shea (2002)). While this literature centered around whether sectoral shocks would translate into aggregate shocks, it has also highlighted that sectoral shocks are important in explaining aggregate fluctuations (Horvath (2000)). The triggering event of the recent crisis in the US subprime market can also be interpreted as a negative sectoral productivity shock as real returns on invested capital in the housing market declined. This sectoral shock has spilled over to other sectors in a dramatic way (see e.g. Shleifer and Vishny (2010)).

This literature has also identified the mechanisms by which shocks in one sector spill over to other sectors and the degree of factor substitutability has turns out to be crucial (Dupor (1999), Horvath (2000)).

In our paper shocks to one sector spill over to the rest of the economy because banks reallocate capital across sectors, as discussed in the next subsection.

2.2 Capital Reallocation and its Limits

An important line of research has documented that the amount of reallocation of existing capital is considerable. Eisfeldt and Rampini (2006) indicate that reallocation of existing capital comprises about one quarter of total investment. In parallel to the empirical literature several theories have been offered why reallocation of capital to its most productive use is impeded and thus may be suboptimal. Apart from physical reallocation costs, informational or contractual

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2 Another line of research has developed models in which there are strategic complementarities across firms so that shocks to some firms can induce cascade effects (Jovanovic (1987) and Durlauf (1993)). Gabaix (2009) shows that idiosyncratic firm-level fluctuations can explain part of aggregate shocks if the firm size distribution is fat tailed. Acemoglu, Ozdaglar and Tahbaz-Salehi (2010) characterize the conditions under which small shocks can create cascade effects in supply networks.

3 They also establish that the reallocation of productive assets across firms is procyclical while the benefits of reallocation are countercyclical. They conclude that the cost or frictions involved in reallocating capital are countercyclical.

4 Earlier studies have found similar magnitudes (Ramey and Shapiro (1998), Maksimovic and Phillips (2001)). Caballero and Hammour (2005) examine whether reallocation shocks lead to lower aggregate output.
frictions have been studied. Eisfeldt (2004) shows that adverse selection in the market for existing assets reduces reallocation, in particular in bad times. Eisfeldt and Rampini (2010) examine a model in which managers have private information about the productivity of capital under their control. Reallocation requires paying large bonuses to unproductive managers in order to reveal the productivity to enable value-increasing reallocation. In particular in bad times this may be too costly to investors and the investor may forgo reallocation. Another channel of capital mobility limits has been identified by Azariadis and Kaas (2009) who focus on limited enforcement of loans when borrowers can default but in such cases are denied access to future loans.

We focus on capital reallocation across sectors. The extent of capital reallocation in our model depends on the degree of substitutability, captured through adjustment costs, on the severity of moral hazard in banks and on the exposure of the banking system to aggregate shocks. We obtain excessive capital mobility. The intuitive reason is pecuniary externalities that are detailed in the next subsection.

2.3 Pecuniary Externalities and Financial Fragility

Our paper is part of a growing literature that highlights the role of pecuniary externalities in generating excessive phenomena in financial markets. Welfare reducing pecuniary externalities occur when agents facing credit frictions act atomistically and do not internalize market price reactions which a social planner facing the same credit frictions would take into account.

The main focus of the recent literature has been on overborrowing and insufficient insurance. In Caballero and Krishnamurthy (2003) and Lorenzoni (2008) entrepreneurs invest too much because they cannot insure against the risk of future binding constraints. In Korinek (2011), financially constrained bankers take on insufficient insurance against binding future constraints as insurance has to be bought from risk-averse households which make it costly. Bianchi (2011) provides a quantitative assessment of macroeconomic and welfare implications of overborrowing and allows for the evaluation of the benefits of policy measures to correct these externalities.

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5Shleifer and Vishny (1992) examine how expected values of assets impact on the debt capacity of a firm.

6Suarez and Sussman (1997) develop a dynamic extension of the Stiglitz-Weiss model of lending under moral hazard. They establish a revision mechanism that also relies on pecuniary externalities. In booms firms start producing more which decreases prices which, in turn, creates liquidity shortages next period. As a consequence, the propensity to default raises and the economy turns into a bust.
Lack of insurance does not play a role in our model. The intuitive reason for the excess volatility of bank lending is as follows. When investment return prospects for all banks become more favorable, they buy additional capital. The opposite occurs when negative aggregate events make selling capital more attractive for all banks.

2.4 Financial Intermediaries and Macroeconomic Shocks

The role of the financial sector and its potentially amplifying impact on business cycle fluctuations has been an enduring theme in economics over the last decades. Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and others have focused on credit constraints faced by non-financial borrowers and have provided conceptual foundations for the so called financial accelerator. The idea is that credit constraints arising from asymmetric information between borrowers and lenders amplify and increase the persistence of even small and transitory exogenous shocks.

The role of the balance sheet of financial intermediaries in amplifying macroeconomic shocks has long been recognized in the empirical macroeconomic literature\(^7\). The empirical literature has also stressed that well-capitalized banks can better absorb macroeconomic shocks\(^8\). Typically, the volatility of bank lending is much higher than the volatility of GDP. For instance, Meh and Moran (2010) report that bank lending growth is over four times as volatile than GDP growth in the US. Adrian and Shin (2010) find that leverage of investment banks is strongly procyclical.

Our analysis provides a rationale for why the volatilities of bank lending and capital prices are excessive and how these volatilities are affected by the characteristics of banks. In particular, the more severe the moral hazard problem in banking is, the higher is the volatility of bank lending. Moreover, higher inside bank equity capital ex ante in the economy reduces the fluctuations of bank financing ex post and smooths macroeconomic shocks. Higher inside equity capital in the economy reduces the willingness of bankers to decrease banks excessively in downturns to attract financing and thus reduces volatility of bank lending.

\(^7\)See e.g. Bernanke and Lown (1991) and Peek and Rosengren (1995).

\(^8\)Theoretical foundations have been rare in the previous century. In recent years, however, a flourishing literature has identified the ways in which banks’ balance sheets transmit aggregate shocks. An entire strand of DSGE modelling frameworks which we cannot summarize in this paper including the banking sector has been developed to quantify the mutual feedbacks between the financial health of banks and real economic activity. A canonical framework of how financial intermediation interact with aggregate economic activity and part of the literature is given in Gertler and Kiyotaki (2011).
3 The Model

We consider a simple three-period economy \((t = 0, 1, 2)\). Initially there is a single physical good that can be transformed into capital at \(t = 0\). It can also be consumed at \(t = 0\) and \(t = 2\). The total amount of physical good that is available in the economy in \(t = 0\) is normalized to 1. The consumption good at \(t = 0\) is taken as a numeraire. There are three classes of agents: bankers, entrepreneurs and investors. All agents live for three periods from \(t = 0\) to \(t = 2\). They are risk-neutral and can consume in \(t = 0\) and \(t = 2\). They do not discount future consumption. The details of the model are set out in the next subsections.

3.1 Agents

There is a continuum of bankers with measure 1. Each banker is endowed with some amount \(e\) of the good (his “wealth”) which varies across bankers. The aggregate endowment of bankers in \(t = 0\) is denoted by \(E\) with 0 < \(E\) < 1.

There is a continuum of investors with measure 1. Aggregate endowments of investors are given by \(1 - E\) as total endowments in the economy are normalized to 1. Finally, there is a continuum of entrepreneurs with measure 1. They only play a passive role in our model.

Because of risk neutrality, social welfare is simply measured by aggregated expected consumption \(U = C_0 + \mathbb{E}(C_2)\).

3.2 Technologies

The model encompasses two sectors or technologies, denoted by FS (the frictionless sector) and BS (the banking sector), respectively. Investments in the FS and the BS entail the formation of a capital good that can be used for production of the consumption good that becomes available at \(t = 2\).

In the BS there is a constant returns technology (the banking technology) that is subject to macroeconomic risk. Specifically, if an amount \(k\) is invested by a bank in \(t = 0\), the output in \(t = 2\) is \(\eta \tilde{R} k\), where \(\tilde{R}\) is an idiosyncratic return and \(\eta\) is a macroeconomic shock with

\[
\eta = \begin{cases} 
  h & \text{(high)} \quad \text{with} \quad \text{prob.} \quad q \\
  l & \text{(low)} \quad \text{with} \quad \text{prob.} \quad 1 - q 
\end{cases}
\]  

(1)
whereby \( q \) and \( 1 - q \) are the probabilities of high and low productivity shocks, respectively, and \( l \) and \( h \) are real numbers that satisfy \( 0 < l < 1 < h \). We denote by \( \tilde{R} \) the expectation of the idiosyncratic return \( \tilde{R} \), which is i.i.d. across banks. The expected output in \( t = 2 \) per unit of investment in the BS is thus \( m \tilde{R} \), where \( m = qh + (1 - q)l \). Without loss of generality, \( m \) is normalized to one. The uncertainty about the aggregate shock is resolved in \( t = 1 \), where all the market participants observe \( \eta \) and learn whether it is high or low.

The technology of the FS exhibits decreasing marginal returns at the aggregate level. Specifically, if an amount \( X \) is invested in period \( t = 0 \) the output in \( t = 2 \) is \( F(X) \) with \( F(0) = 0 \), \( F'(X) > 0 \) and \( F''(X) < 0 \). \( F(\cdot) \) is assumed to fulfill the Inada conditions, i.e. \( \lim_{X \to 0} F'(X) = \infty \) and \( \lim_{X \to 1} F'(X) = 0 \). These two conditions ensure that some but not all of the resources are invested in the FS in \( t = 0 \). Note that the technology shock is sectoral: it only impacts the sector financed by banks. The analysis could be easily extended to technology shocks that impact both sectors.\(^9\)

### 3.3 Entrepreneurs

Entrepreneurs operate the technologies but they only play a passive role in our model. Those operating the frictionless technology are directly financed by investors. Those operating the banking technology must be monitored, and therefore have to be financed by banks.\(^{10}\) Because our focus is on the macroeconomic impact of shocks to the banking sector, we do not model entrepreneurs explicitly. However, as we assume perfectly competitive markets in the FS, it is useful to think of entrepreneurs in this sector as being a continuum of agents. Each agent has access to an invisible project of size one that delivers consumption good \( x \) in \( t = 2 \). The productivity \( x \) is distributed according to some continuous and differentiable distribution function \( G(x) \) on \([0, \infty)\). Then, if an amount \( X \) is invested in the sector, the marginal investor with productivity \( \tilde{x} \) who just receives funds is given by

\[
X = \int_{\tilde{x}}^{\infty} dG(x).
\]

Total output is given by

\[
F(X) = \int_{\tilde{x}}^{\infty} x dG(x)
\]

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\(^9\)In such circumstances, no capital reallocation may occur in the frictionless economy. In the presence of moral hazard and banks, however, inefficient capital reallocation takes place.

\(^{10}\)The costs of monitoring are set to zero.
and the marginal productivity is
\[ F'(X) = \tilde{x}. \]

### 3.4 Capital Allocation

In period \( t = 0 \) some amount \( C_0 \) of the good is consumed and the rest is transformed into capital and allocated between the two technologies: \( K \) to the banking technology and \( X \) to the frictionless technology. The aggregate resource constraint amounts to \( C_0 + K + X = 1 \). Upon observing macroeconomic events in \( t = 1 \) the scale of investments in the BS and the FS can be adjusted by reallocating capital between the two sectors. We denote by \( p_\eta \) the price for 1 unit of capital used at date 1 in the FS in state \( \eta \), in terms of claims on consumption good at \( t = 2 \). The price \( p_\eta \) is the interim rate of return on capital in the FS.

As of date 0, the (ex-ante) rate of return on capital is \( \mathbb{E}[p_\eta] = 1 + r \). \( r \geq 0 \) can be interpreted as the interest rate. Note that, given risk neutral preferences with no discounting, either \( r \) or \( C_0 \) must be zero.

Capital is traded against claims on future consumption at \( t = 2 \). There are no defaults nor contract enforcement problems. In period \( t = 2 \) no further trade takes place. The consumption good at \( t = 0 \) is taken as a numéraire. We assume that financial markets are complete and frictionless. Given risk neutral preferences, the contingent price paid at \( t = 0 \) against delivery of the good at \( t = 2 \) in state \( h \) (resp. \( l \)) is simply equal to the discounted probability \( \frac{q}{1+r} \) (resp. \( \frac{1-q}{1+r} \)) of this state, where \( r \geq 0 \) is the interest state. In all interior allocations \( 0 < C_0 < 1 \) the interest rate is necessarily zero.

At the interim period \( t = 1 \) capital goods are sold or bought by bankers and entrepreneurs. We assume that investments in the FS are observable. Hence, claims on investment returns in the frictionless technology can be used as a means of payment in the market for capital goods in \( t = 1 \). The rate of return \( p_\eta \) thus represents the amount of consumption good at \( t = 2 \) that is exchanged for one unit of capital at \( t = 1 \) in state \( \eta \).

### 3.5 Banks

Each banker faces the sequence of decisions and events illustrated in Figure 1.

We denote by \( k_\eta = (1 + \alpha_\eta)k \) the capital invested by the typical bank after the adjustment decision, where \( \alpha_\eta \) depends on the macroeconomic shock \( \eta \) and satisfies \( \alpha_\eta \geq -1 \). A value
Banker has endowment $e$ and borrows $k-e$ from investors according to the contract $C(k, \alpha \eta, b \eta)$. Banker invests $k$ in the BS (size of the bank) and macroeconomic shock $\eta$ occurs and is publicly observed. Bank adjusts its size by a fraction $\alpha \eta$. Bank size becomes $k_\eta = (1 + \alpha \eta)k$. Capital is sold or bought at price $p_\eta$. Moral hazard: Banker exerts effort (successful outcome $o=S$ with probability $\tau$, no private benefit) or shirks (success with prob: $\tau-\Delta$, private benefit $B k_\eta$). Outcome: $R_\eta k_\eta$ if success and $\eta$ has occurred. Payment to banker: $b \eta k_\eta$. Investors get the rest. $S_\eta o \eta o \eta t = 0 t = 1 t = 2$

$\alpha \eta > 0$ characterizes additional investment in the BS and $\alpha \eta < 0$ expresses disinvestments. We can interpret $\alpha \eta = \frac{k_\eta-k}{k}$ as the growth of credit in the BS. Capital can be bought or sold for a promised repayment $p_\eta$. Moreover, investment adjustments involve additional costs $\frac{c}{2} \alpha^2 \eta k$ where $c$ is a positive constant that measures the relative ease of reallocating capital across sectors. Adjustment costs reduce output in the BS and thus are incurred at $t = 2$, and are deducted from gross returns on investment.

The banker’s investments are subject to moral hazard (see Holmström and Tirole (1997)). The project outcome is either a success ($o=S$) or a failure ($o=F$), and therefore the idiosyncratic return $\tilde{R}$ is either $R^S$ or $R^F$. If the banker exerts effort, or equivalently, if he chooses a project with high probability of success, he has no private benefit and $o=S$ occurs with probability $\tau$. If the banker shirks or equivalently chooses a project with lower prospects of success, he receives a private benefit $Bk_\eta > 0$ and $o=S$ occurs with probability $\tau - \Delta$. $B$ is measured in terms of the consumption good. The banker receives a payment $b_\eta \eta k_\eta$ when the macroeconomic shock $\eta$ and the project outcome $o$ have occurred. The contract between the banker and investors is denoted by $C(k, \alpha \eta, b_\eta \eta)$. The timeline of investment and returns is summarized in Figure 1.

Figure 2 represents the random structure of returns. The capital adjustment and reallocation decisions are made by each banker after the realization of the macro shock. The probability of success is $\tau$ when the banker exerts effort, and only $\tau - \Delta$ when the banker shirks. The term in brackets represents the impact of shirking on the probabilities of success and failure.
3.6 Social Welfare

In this risk neutral economy, social welfare is measured by aggregate expected consumption:\(^\text{11}\)

\[ U = C_0 + E[C_2] = 1 - K - X + KE[(1 + \alpha_\eta)\eta \bar{R} - \frac{c}{2}\bar{\alpha}^2_\eta] + E[F(X - \alpha_\eta K)] \]

This aggregate consumption is shared across agents according to the following aggregate rules:

- Investors get \((1 + r)(1 - E)\), where \(r\) is the interest rate.
- Bankers get \(KE[b_\eta^*(1 + \alpha_\eta)]\).
- Entrepreneurs get \(E[F(X - \alpha_\eta K) - p_\eta(X - \alpha_\eta K)]\), where \(p_\eta = F'(X - \alpha_\eta K)\) is the marginal productivity of capital in the FS and thus the price of capital in state \(\eta\).

At equilibrium, the sum of these three terms coincides with \(U\). This is because the expected rate of return on capital in the banking sector has to be equal to \(1 + r\), as the risk neutral investors have to be indifferent between investments in the BS and the FS, i.e.:

\[ (1 + r)(K - E) = KE[(1 + \alpha_\eta)(\eta \bar{R} - b_\eta^*) - \frac{c}{2}\bar{\alpha}^2_\eta - p_\eta \alpha_\eta]. \]

\(^{11}\)In general, the private benefits of bankers also enter social welfare. Throughout the paper, we will focus on circumstances in which shirking in inefficient and will be avoided by paying the banker a higher amount if the project outcome is a success.
$K - E$ is the amount of resources offered by investors to the BS. Therefore \(^{12}\)

$$U = (1+r)C_0 + \mathbb{E}[C_2] = (1+r)(1-K-X)+(1+r)(K-E)+K\mathbb{E}[b_\eta^0(1+\alpha_\eta)+p_\eta\alpha_\eta]+\mathbb{E}[F(X-\alpha_\eta K)],$$

and after simplification,

$$U = (1 + r)(1 - E) + K\mathbb{E}[b_\eta^0(1 + \alpha_\eta)] + \mathbb{E}[F(X - \alpha_\eta K) - p_\eta(X - \alpha_\eta K)],$$

where we have used the fact that \(\mathbb{E}[p_\eta] = 1 + r\). This decomposition will reveal useful for policy discussions.

### 4 Two Simple Benchmarks

This section examines two cases where public intervention is not justified: the frictionless economy (no moral hazard) and the rigid economy (no capital reallocations).

#### 4.1 The Frictionless Economy

In the absence of moral hazard, which is equivalent to setting \(B = 0\), supply and demand for capital in both sectors, the BS and the FS, are not subject to frictions. The competitive equilibrium of this economy leads to a first best social optimum. To establish uniqueness and social efficiency of the competitive equilibrium, we note first that the rate of return on capital in state \(\eta\) in \(t = 1\) must be equal to the marginal return of investment in both technologies:

$$p_\eta = F'(X - K\alpha_\eta) = \eta\bar{R} - c\alpha_\eta. \quad (2)$$

Moreover, the investment decisions at date 0 are determined by the equality of expected returns on capital in these two technologies:

$$\mathbb{E}[p_\eta] = \mathbb{E}[\eta\bar{R}(1 + \alpha_\eta) - \frac{C}{2}\alpha_\eta^2 - \alpha_\eta p_\eta].$$

Combining these two properties, we see that

$$\mathbb{E}[\eta\bar{R} - c\alpha_\eta] = \mathbb{E}[\eta\bar{R}(1 + \alpha_\eta) - \frac{C}{2}\alpha_\eta^2 - \alpha_\eta(\eta\bar{R} - c\alpha_\eta)].$$

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\(^{12}\)Recall that \(rC_0 = 0\) (either \(r = 0\) or \(C_0 = 0\)) by the complementarity slackness condition.
which leads, after simplification, to a fundamental relation between \( \alpha_h \) and \( \alpha_l \), namely:

\[
E[\alpha_h^2 + 2\alpha_l] = 0. \tag{3}
\]

This condition will play an important role in the sequel, as it is also satisfied in the competitive equilibrium of the economy with moral hazard and capital reallocations. It characterizes a decreasing curve in the \((\alpha_l, \alpha_h)\) plane, limited by two extreme points:

- \((-1, \frac{1}{\sqrt{q}} - 1)\), which corresponds to the complete liquidation of banking assets in the bad state,\(^{13}\) and
- \((0, 0)\), which corresponds to a complete absence of adjustment at date 1.

It is easy to see that this curve can be parameterized by the variance of \( \alpha \), namely \( \sigma^2 = q(1 - q)(\alpha_h - \alpha_l)^2 \). Indeed condition (3) is equivalent to:

\[
\sigma^2 + \bar{\alpha}^2 + 2\bar{\alpha} = 0,
\]

with \( \bar{\alpha} := E[\alpha] \). This implies\(^{14}\) that \( \bar{\alpha} = \sqrt{1 - \sigma^2} - 1 \) and

\[
\begin{align*}
\alpha_h &= \sqrt{1 - \sigma^2} - 1 + \sigma \sqrt{\frac{1 - q}{q}} \\
\alpha_l &= \sqrt{1 - \sigma^2} - 1 - \sigma \sqrt{\frac{q}{1 - q}}
\end{align*}
\]

To determine the first best allocation, we need a second condition, that is given by the equality of supply and demand for capital in the banking sector. It is natural to focus on the case where the banking technology is profitable even when there are no capital reallocations, namely:

**Assumption 1**

\( \bar{R} > 1 \).

Under assumption 1, the first best allocation involves no consumption at \( t = 0 \) (\( C_0 = 0 \), and \( K + X = 1 \)) and a positive interest rate \( r \), that can be deduced from condition (2), transformed by using the fact that \( X = 1 - K \):

\[
1 + r = E \left[ F'[1 - K(1 + \alpha)] \right] = E \left[ \eta \bar{R} - c\alpha \right]. \tag{4}
\]

\(^{13}\)In order to ensure that output in the BS is non-negative we can restrict \( \alpha_l \geq \alpha \), \( \alpha \) is defined by \( l(1 + \alpha)k - \frac{1}{2}k^2 = 0 \), which yields \( \alpha = \frac{l + \sqrt{l^2 + 2cl}}{c} \).

\(^{14}\)The second solution of equation (3) is not applicable as it violates the boundary condition of \( \alpha \).
Let us define the supply of capital to the banking sector as the function \( S(p) \) such that

\[
F'[1 - S(p)] = p. \tag{5}
\]

\( S(\cdot) \) is an increasing function such that \( S(0) = 0 \) and \( S(\infty) = 1 \). Condition (4) just means that:

\[
K(1 + \alpha_\eta) = S(\eta \bar{R} - c\alpha_\eta),
\]

which gives the second condition that we need to characterize the first best allocation:

\[
\frac{S(h \bar{R} - c\alpha_h)}{1 + \alpha_h} = \frac{S(l \bar{R} - c\alpha_l)}{1 + \alpha_l}. \tag{6}
\]

**Proposition 1**

*In the frictionless economy there is a unique competitive equilibrium characterized by conditions (3) and (6). This competitive equilibrium is Pareto optimal.*

Proposition 1 follows from the properties of the function \( S(p) \) and the first welfare theorem.

**4.2 The Rigid Economy**

As a second benchmark, let us consider the case where capital adjustments are impossible \( (c = \infty) \) but moral hazard is present. We will show that, in this case also, the competitive allocation cannot be Pareto improved and that public intervention is not needed.

In fact, macroeconomic shocks do not play any role in this rigid economy. Since agents are all risk neutral and no adjustments can be made at \( t = 1 \), the macroeconomic shock has no impact on utilities. The rigid economy is thus a simple extension of the static model of Holmström and Tirole (1997). We assume that investment in the BS is socially beneficial only if the banker exerts effort:

**Assumption 2**

\[
\bar{R} + B - \Delta(R^S - R^F) < 1
\]
Note that Assumptions 1 and 2 taken together imply that $R_S - R_F > \frac{B}{\Delta}$. Effort is exerted if and only if payments to the banker satisfy

$$\Delta(b^S_\eta - b^F_\eta)k \geq kB$$

or $b^S_\eta - b^F_\eta \geq \frac{B}{\Delta}$. \hfill (7)

Since the banker is protected by limited liability ($b^o_\eta \geq 0$), $o = S, F$, it is easy to see the optimal contract is such that $b^S_\eta = \frac{B}{\Delta}$, $b^F_\eta = 0$ for $\eta \in \{h, l\}$. Under Assumption 2, the optimal contract between investors and the banker consists of paying him a bonus $\frac{B}{\Delta}k$ (proportional to the size $k$ of the bank) in case of success, and nothing in case of failure. This contract minimizes the informational rents of the banker. The optimal size of the bank is the maximum $k$ that satisfies the break-even constraint of investors:

$$(k - e)(1 + r) \leq k(\bar{R} - b),$$

where $b = \frac{\tau B}{\Delta}$ represents the expected payment to the banker per unit of asset. This condition is equivalent to a (market imposed) capital ratio\textsuperscript{15}.

$$\frac{e}{k} \geq \rho_0(r),$$

where the capital ratio $\rho_0(r) = 1 - \frac{\bar{R} - b}{1 + r}$ is an increasing function of $r$.

Note that this contract can be implemented by a simple capital structure whereby the bank issues riskless deposits with total repayment $D_R = lkR_F$ in $t = 0$ (corresponding to the minimum liquidation value of the bank’s assets) and additional debt $\Delta D_R$ with total repayment $k(hR_F - lR_F)$ in $t = 1$ if $\eta = h$ has occurred. Moreover, the bank issues (outside) equity in $t = 0$ that pays out a dividend $k[\eta R_S - \eta R_F - \frac{B}{\Delta}]$ in case of success. The banker’s payment $\frac{kb}{\eta}$ is only paid in case of success. An alternative is to give the banker some (inside) equity. In this case, total dividend payments amount to $k\eta[R^S - R^F]$ of which a fraction $\theta_\eta = \frac{B}{\Delta \eta(R^S - R^F)}$ is given to the banker as inside equity in $t = 1$. Assumption 2 implies that $\theta_\eta \leq 1$.

The equilibrium interest rate is then given by the equality of supply and demand for banking capital:

$$K = \frac{E}{\rho_0(r)} = S(1 + r).$$

\textsuperscript{15}Our banks are financed by sophisticated investors, not by uninformed retail depositors. Thus there is no need for micro-prudential regulation.
When $E$ is small (specifically $E < E^* := S(1)\rho_0(0)$) this equilibrium interest rate is zero and $C_0 > 0$. When $E$ is large (specifically $E > E^{**} = \frac{b}{R}S(\bar{R})$) the moral hazard constraint ceases to bind and social welfare is maximal. Social welfare in this case is given by:

$$U = E(C_2) = \max_X \{ F(X) + (1 - X)\bar{R} \}$$

where $K = 1 - X = S(\bar{R})$.

Note that, when $E \leq E^{**}$, social welfare could be increased by transferring wealth ex-ante from investors to bankers. We will systematically rule out such transfers because they would require giving taxation powers to banking supervisors, which would face strong political opposition. Instead we will focus on traditional regulatory instruments such as capital or liquidity ratios that do not involve payment flows between supervisors and banks. We summarize the three possible cases in this section in the following proposition.

**Proposition 2**

Assume $\bar{R} < 1 + b$. Then the rigid economy has a unique equilibrium, characterized as follows:

- When $E < E^* = S(1)\rho_0(0)$: $K = \frac{E}{\rho_0(0)}$, $X = 1 - S(1) - C_0$, $C_0 > 0$ and $r = 0$.

- When $E^* \leq E \leq E^{**} = \frac{b}{R}S(\bar{R})$: $K = \frac{E}{\rho_0(r)} = S(1 + r)$, $C_0 = 0$.

- When $E > \frac{b}{R}S(\bar{R})$: $K = S(\bar{R}) = 1 - X$, $C_0 = 0$

As final comments to this section, note that:

- There is no need for maturity transformation: bankers find it optimal to choose the maturity of their debt to be equal to the maturity of their assets ($t = 2$),

- Any regulatory intervention on the size of banks would be socially wasteful: reducing the size of banks would decrease social surplus, while forcing them to lend more would generate shirking.

5 Privately Optimal Contracts

5.1 The Contractual Problem

In the economy with capital adjustments, contracts between each banker and its financiers have to specify $\alpha_h$ and $\alpha_l$. The optimal payments to the banker follow the same logic as in the rigid economy, with the only difference that they will in general depend on the macro shock
η. Assuming for instance that private benefits per unit of assets (obtained by the banker who shirks) are still $B$ (independently of $\eta$), the optimal payments to the banker are $\frac{B}{\Delta} k\eta$ in case of success (and zero in case of failure). This means that the expectation of the rent that investors have to give to the banker is $b \mathbb{E} k\eta = bk\mathbb{E}(1 + \alpha\eta)$, where $b = \frac{\tau B}{\Delta}$. Thus the optimal banking contract solves

$$\max_{k,\alpha_h,\alpha_l} bk\mathbb{E}(1 + \alpha\eta)$$

subject to the investor’s participation constraint:

$$(k - e)(1 + r) \geq k\mathbb{E}[(1 + \alpha\eta)(\eta\bar{R} - b) - \frac{c}{2}\alpha^2\eta - p\eta\alpha\eta]$$

and the banker’s participation constraint:

$$bk\mathbb{E}[1 + \alpha\eta] \geq e(1 + r).$$

We will focus on cases where bank capital is scarce ($E$ is small) and thus the second constraint does not bind and $C_0 > 0$ which implies $r = 0$. Note that, by absence of arbitrage opportunities, we must have

$$\mathbb{E}[p\eta] = 1.$$

### 5.2 Solution

The following Proposition characterizes the solution of the contractual problem.

**Proposition 3**

*Given a vector $p = (p_h, p_l)$ of capital prices and banker’s initial wealth $e$, the optimal banking contract is characterized by*

$$k(p, e) := \frac{e}{\rho(p)}, \text{ where}$$

$$\rho(p) = 1 - \mathbb{E}[(1 + \alpha_h(p)))(\eta\bar{R} - b) - \frac{c}{2}\alpha^2_h(p) - p_h\alpha\eta(p)]$$

*and $(\alpha_h(p), \alpha_l(p))$ maximize*

$$\frac{b\mathbb{E}[1 + \alpha\eta(p)]e}{1 - \mathbb{E}[(1 + \alpha\eta(p))(\eta\bar{R} - b) - \frac{c}{2}\alpha^2\eta(p) - p\eta\alpha\eta(p)].}$$

Proposition 3 follows directly by the participation constraint of the investor and solving for $k$ with $r = 0$. The solution of the maximization problem will be given in the next section. We
note that adjustment decisions \((\alpha_h(p), \alpha_l(p))\) are independent of the scale of banks. Moreover, the size \(k\) of the bank is proportional to the banker’s wealth \(e\), a property that will make aggregation easy.

Proposition 3 implies that there is a “market-imposed” capital ratio for banks:

\[
\frac{e}{k} \geq \rho(p).
\]

This equity ratio is a function of the capital prices \(p_h\) and \(p_l\).

### 5.3 Implementation of the Optimal Contract

We now show that the optimal banking contract can be implemented by a certain capital structure, involving a combination of short term debt, long term debt and some insurance against aggregate shocks.

The role of short term debt is to “discipline” bankers, in the spirit of Jensen (1983) and Diamond and Rajan (2000). Since their bonus is proportional to the size of their bank, bankers would not spontaneously reduce this size by the factor \(\alpha_l < 0\) in the state \(l\). To force them to do so, investors can impose a debt repayment of an amount equal to the net proceeds of liquidation in this state:

\[
D_{ST} = -k\alpha_l p_l > 0.
\]

Of course, this short term debt increases the needs for funds in the good state \(h\). In that state the bank must then be allowed to issue new debt for the amount:

\[
\Delta D = D_{ST} + k\alpha_h p_h.
\]

This particular debt structure is needed to force the bankers to implement the reallocation decisions \((\alpha_l, \alpha_h)\) that are optimal for investors. There is some degree of freedom for the rest of the implementation since the Modigliani-Miller logic applies to the total payments that investors must receive\(^\text{16}\) at date 2. When state \(\eta\) has occurred, these payments are:

\[
\begin{align*}
P^S_\eta &= k(1 + \alpha_\eta)[\eta R^S - \frac{B}{\Delta} - \frac{c}{2}\alpha_\eta^2 k] \quad \text{in case of success, and} \\

P^F_\eta &= k(1 + \alpha_\eta)[\eta R^F - \frac{c}{2}\alpha_\eta^2 k] \quad \text{in case of failure},
\end{align*}
\]

\(^\text{16}\)The contract only specifies the payments to the bankers. The payments to investors can be split arbitrarily between different types of securities.
while the banker receives

\[ b_S^F = k(1 + \alpha) \frac{B}{\Delta} \quad \text{in case of success, and} \]
\[ b_F^F = 0 \quad \text{in case of failure.} \]

A natural choice of securities that implements these payments is a combination of debt and equity such that dividends are paid only in case of success. Total debt repayment is then, \( P^F \), while dividends \( \delta = P^S - P^F \) are paid in case of success. The banker receives a bonus \( k(1 + \alpha) \frac{B}{\Delta} \) in case of success and nothing in case of failure. Note that the total repayment to initial debtholders is not the same in the two states. In state \( l \), this total repayment is \( P^F_l = k(1 + \alpha)lR^F - \frac{\alpha_l}{2}k \). This is the amount that can be paid back in any case and thus can be promised as repayment for a long-term debt, issued in \( t = 0 \) and repaid in \( t = 2 \). The latter amount is \( D_{LT} \). However if state \( h \) occurs, new debt \( \Delta D \) is issued at date 1 and the borrowing capacity of the bank increases, so that total repayment to initial debtholders at date 2 in case of failure becomes:

\[ P^F_h - \Delta D = k \left[ (1 + \alpha)hR^F - \alpha_hp_l - \frac{c}{2}(\alpha_h)^2 + \alpha_l p_l \right]. \]

In general, this amount is higher than \( P^F_l \). Thus, some insurance against macro shocks must be included in the package of securities that implements the optimal banking contract. One example of such an insurance mechanism is a convertible bond with a face value \( C^N = (P^F_h - \Delta D) - P^F_l \) that has to be repaid if it is not converted to equity. If state \( l \) occurs, this bond is converted into equity, so that total repayment to debtholders is reduced to \( P^F_l \), which corresponds to the volume of long term debt issued at \( t = 0 \).

To wrap up, the competitive contract can be implemented as follows:

- at date 0, the bank issues equity for an amount \( E \), short term debt for an amount \( D_{ST} \), long term debt for an amount \( D_{LT} \) and convertible bonds for an amount \( C^N \).
- if state \( l \) occurs, the bank liquidates at date 1 a fraction \( |\alpha_l| \) of its assets, and uses the net proceeds to repay its short term debt:

\[ D_{ST} = -k\alpha_l p_l > 0. \]

Each convertible bond is converted into \( y \) shares. At date 2, long term debt \( D_{LT} = k(1 + \alpha)lR^F - \frac{\alpha_l}{2}(\alpha_l)^2k \) is repaid and in case of success, a total amount of dividends \( k [1 + \alpha_l][l(R^S - R^F) - \frac{B}{\Delta}] \) is paid to initial shareholders, and new shareholders (who initially held convertible debt). In case of success, the banker receives \( k(1 + \alpha) \frac{B}{\Delta} \).
• if state $h$ occurs, the bank issues new debt for an amount $\Delta D = D_{ST} + k \alpha_h p_h$. It increases its size by a factor $\alpha_h$. At date 2, total payment to debt holders is

$$P^F_h = D_{LT} + \Delta D + C^N = k(1 + \alpha_h)hR^F - \frac{c}{2}(\alpha_h)^2 k$$

where

$$C^N = k \left[ (1 + \alpha_h)hR^F - (1 + \alpha_l)lR^F - \alpha_hp_h - \frac{c}{2}(\alpha_h)^2 + \alpha_lp_l + \frac{c}{2}(\alpha_l)^2 \right]$$

is the face value of the convertible bond.

Finally, if the bank succeeds, the banker receives a bonus $k(1 + \alpha_h)\frac{B}{\Delta}$ and shareholders receive dividends $k[1 + \alpha_h] \left[ h(R^S - R^F) - \frac{B}{\Delta} \right]$.

6 Competitive Equilibrium

6.1 Definition

In this section we characterize the competitive equilibrium of the economy. For this purpose we recall that we have denoted by $C_0$ the aggregate consumption at $t = 0$, by $K$ the aggregate amount of capital invested in $t = 0$ in the BS and by $X$ the aggregate amount invested in the FS in $t = 0$. Henceforth, the variables $\alpha_h$, $\alpha_l$ and $k$ always correspond to the privately optimal values, given by Proposition 3.

Definition 1

A competitive equilibrium in the economy with moral hazard and capital reallocation is an array $\Sigma = \{C_0, K, X, \alpha_h, \alpha_l, p_h, p_l\}$ such that:

(i) Each banker with wealth $e$ obtains $k - e$ with

$$k = \frac{e}{\rho(p)}$$

Aggregate investments in the BS in $t = 0$ are

$$K = \frac{E}{\rho(p)}$$

(ii) Investments in the BS are adjusted in $t = 1$ by $\alpha_h(p), \alpha_l(p)$ defined in Proposition 3.

(iii) The rate of return on capital at $t = 1$ in state $\eta$ is $p_\eta = F'(X - \alpha_\eta(p)K), \quad \eta \in \{h, l\}$
(iv) $E[p_\eta] = qp_h + (1 - q)p_l = 1 + r$.

(v) $K + X + C_0 = 1$.

(vi) $C_0 \geq 0$, $K \geq 0$, $X \geq 0$.

(vii) $rC_0 = 0$.

Thus at date 0, $K$ is invested in the banking technology, $X$ in the frictionless technology and $C_0 = 1 - K - X$ is consumed. At date 1, the aggregate shock $\eta$ is revealed, and investments are adjusted to $K_\eta = K(1 + \alpha_\eta)$ and $X_\eta = X - \alpha_\eta K$. The rate of return on capital is determined by its marginal productivity in the frictionless technology. We will focus on interior solutions, in which $C_0 > 0$ and the expected rate of return on capital in equilibrium is equal to 1 (i.e. $r = 0$).

Finally, at date 2, the average return on bank assets is $\eta R$, while the frictionless sector produces $F(X_\eta)$. Aggregate output in state $\eta$ is thus:

$$Y_\eta = \eta R K_\eta + F(X_\eta) - \frac{c}{2} \alpha_\eta^2 K.$$  \hspace{1cm} (12)

6.2 Existence and Uniqueness

To obtain a handy characterization of the competitive equilibrium (and an existence theorem) let us reformulate Proposition 3. Because the banking technology has constant returns to scale, the optimal contract can be characterized at the aggregate level. That is, $(K, \alpha_h, \alpha_l)$ is the solution of

$$\max_K bE[1 + \alpha_\eta]$$ \hspace{1cm} (13)

subject to

$$KE[(1 + \alpha_\eta)(\eta R - b) - \frac{c}{2} \alpha_\eta^2 - p_\eta \alpha_\eta] = K - E.$$  \hspace{1cm} (14)

Denoting by $L$ the associated Lagrangian, and by $\nu$ the multiplier, we can characterize this solution by the first order conditions:

$$\frac{\partial L}{\partial K} = bE[1 + \alpha_\eta] + \nu E[(1 + \alpha_\eta)(\eta R - b) - \frac{c}{2} \alpha_\eta^2 - p_\eta \alpha_\eta] - \nu = 0$$ \hspace{1cm} (14)

$$\frac{\partial L}{\partial \alpha_\eta} = K Pr(\eta)[b + \nu(\eta R - b - c\alpha_\eta - p_\eta)] = 0,$$ \hspace{1cm} (15)
where $Pr(\eta)$ denotes the probability that $\eta$ occurs, which is $q$ for $\eta = h$ and $1 - q$ for $\eta = l$. From the second condition we derive:

$$ p_\eta = \eta \bar{R} - b - c\alpha_\eta + \frac{b}{\nu}. \tag{16} $$

Plugging this expression into the first condition yields:

$$ 1 - \frac{b}{\nu}(1 + \bar{\alpha}) = E[(\eta \bar{R} - b)(1 + \alpha_\eta) - \frac{c}{2} \alpha_\eta^2 - \alpha_\eta(\eta \bar{R} - b - c\alpha_\eta + \frac{b}{\nu})], \tag{17} $$

with $\bar{\alpha} = \mathbb{E}[\alpha_\eta]$. After simplifications we obtain:

$$ \frac{b}{\nu} = 1 - \bar{R} + b - \frac{c}{2} \mathbb{E}[\alpha_\eta^2], \tag{18} $$

and thus\(^{17}\)

$$ p_\eta = (\eta - 1)\bar{R} + 1 - c\alpha_\eta - \frac{c}{2} \mathbb{E}[\alpha_\eta^2]. \tag{19} $$

Now the participation constraint of investors gives

$$ K - E = K[1 - \frac{b}{\nu}(1 + \bar{\alpha})], \tag{20} $$

or

$$ \frac{E}{K} = (1 + \bar{\alpha})[1 - \bar{R} + b - \frac{c}{2} \mathbb{E}[\alpha_\eta^2]] \tag{21} $$

Now the first equilibrium condition gives:

$$ \mathbb{E}[p_\eta] = 1. \tag{22} $$

Using the expression of $p_\eta$ obtained above we get

$$ 1 = \mathbb{E}[(\eta - 1)\bar{R} + 1 - c\alpha_\eta - \frac{c}{2} \mathbb{E}[\alpha_\eta^2]], \text{ and thus} \tag{23} $$

$$ \bar{\alpha} + \frac{1}{2} \mathbb{E}[\alpha_\eta^2] = 0. \tag{24} $$

As above, it is convenient to parameterize the equilibrium by the variance of $\alpha_\eta$, namely $\sigma^2 = q(1-q)(\alpha_h - \alpha_l)^2$. The above condition implies that $2\bar{\alpha} + \sigma^2 = 0$. Thus $\bar{\alpha} = \sqrt{1 - \sigma^2} - 1$ and thus:

\(^{17}\)The last condition also implies $\nu > 1$. 

21
\[
\begin{align*}
\alpha_h &= \sqrt{1 - \sigma^2} - 1 + \sigma \sqrt{\frac{1-q}{q}} \\
\alpha_l &= \sqrt{1 - \sigma^2} - 1 - \sigma \sqrt{\frac{q}{1-q}}
\end{align*}
\] (25)

The rates of return on capital can be expressed as
\[
\begin{align*}
p_h &= (h-1)R + 1 - c\sigma \sqrt{\frac{1-q}{q}} \\
p_l &= (l-1)R + 1 + c\sigma \sqrt{\frac{q}{1-q}}
\end{align*}
\] (26)

We observe

**Fact 1**

(i) \(-1 < \alpha_l < 0, \alpha_h > 0\)

(ii) \(p_h > 1, p_l < 1\)

Fact 1 follows from formulas (25) and (26). While \(\alpha_l < 0\) follows immediately, verifying \(\alpha_h > 0\) requires manipulation of equation (25) and the use of the fact that \(\alpha_l > -1\).

The final equilibrium conditions are:

\[p_{\eta} = F'(X - \alpha_\eta K), \quad \eta = h, l,\] (27)

where \(K\) is such that

\[\frac{E}{K} = (\sqrt{1 - \sigma^2}[1 - R + b + c\sqrt{1 - \sigma^2} - c].\] (28)

Using the definition of \(S(p)\) in equation (5), and thus \(1 - S(p_{\eta}) = X - \alpha_\eta K\), we can eliminate \(X\) between these conditions gives:

\[(\alpha_h - \alpha_l)K = S(p_h) - S(p_l),\] (29)

where \(S(.)\) is the supply function of bank capital defined above.
This provides the last condition on $\sigma$:

$$
\frac{1}{\sqrt{q(1-q)}} \sqrt{1 - \sigma^2 [1 - \bar{R} + b + c\sqrt{1 - \sigma^2 - c}]} E\sigma
= S \left( (h - 1)\bar{R} + 1 - c\sigma\sqrt{\frac{1 - q}{q}} \right) - S \left( (l - 1)\bar{R} + 1 + c\sigma\sqrt{\frac{q}{1 - q}} \right),
$$

or alternatively

$$
E = \sqrt{q(1-q)} \left( \frac{1}{\sigma^2} - 1 \right) \left[ 1 - \bar{R} + b + c\sqrt{1 - \sigma^2 - c} \right].
\cdot \left\{ S \left( (h - 1)\bar{R} + 1 - c\sigma\sqrt{\frac{1 - q}{q}} \right) - S \left( (l - 1)\bar{R} + 1 + c\sigma\sqrt{\frac{q}{1 - q}} \right) \right\}
$$

The righthand side of this equation is a function of $\sigma$ that we denote $H(\sigma)$. It is the product of three decreasing functions of $\sigma$, which are positive on $(0, 1)$. Therefore $H$ is strictly decreasing on $(0, 1)$. Moreover $H(0) = +\infty$ and $H(1) = 0$. This guarantees the existence of a unique solution $\sigma^E$ to the equation $H(\sigma) = E$. Thus we have established:

**Proposition 4**

There is a unique competitive equilibrium that can be parameterized by the variance of the credit growth: $\sigma^2 = q(1-q)(\alpha_h - \alpha_l)^2$.

- $\sigma^E$ is the unique solution of $H(\sigma) = E$.
- $\alpha^E = \sqrt{1 - \sigma^2} - 1$
- $p^E_h = (h - 1)\bar{R} + 1 - c\sigma^E\sqrt{\frac{1 - q}{q}}$, $p^E_l = (l - 1)\bar{R} + 1 + c\sigma^E\sqrt{\frac{q}{1 - q}}$
- $\frac{E}{K^E} = \sqrt{1 - \sigma^2} [1 - \bar{R} + b + c\sqrt{1 - \sigma^2 - c}]$

Proposition 4 and the properties of equation (31), yield immediately the following Corollary.

**Corollary 1**

When $E$, the capital of the banking industry increases:

- The standard deviation $\sigma^E$ of bank credit growth decreases and its mean $\alpha^E$ increases.
- $p^E_h$ increases and $p^E_l$ decreases (capital prices vary more)
- $\frac{E}{K^E}$ increases.
Another immediate consequence of equation (31) and Proposition 4 is,

**Corollary 2**
An increase of the severity of moral hazard in banking, i.e. an increase of \( b \),
- increases \( \sigma^E \) and lowers \( \alpha^E \)
- decreases \( \alpha_l \) and increases \( \alpha_h \) if \( \sigma \) is not very large
- decreases \( p_h^E \) and increases \( p_l^E \).

### 7 Social Efficiency and Regulation

In this section we explore the scope for regulation.

Given that all agents are risk neutral, social efficiency is simply measured by expected aggregate consumption (remember that we focus on the case where \( r = 0 \)):

\[
U = KE[\eta R(1 + \alpha_\eta) - \frac{C}{2} \alpha_\eta^2 - 1] + E[F(X - \alpha_\eta K)] - X + 1 \tag{32}
\]

**7.1 The Competitive Allocation is Inefficient**

We first show that capital reallocation at the competitive equilibrium is inefficient. In particular, we show in the next proposition that, starting from the competitive equilibrium, characterized in Proposition 4, the expected social surplus \( U \) can be increased by raising \( \alpha_l \) and adjusting \( \alpha_h \). Such an intervention would force banks to invest more during recessions and less in booms if \( \Delta \alpha_h < 0 \). We will later discuss which type of regulatory tools can achieve this objective.

**Proposition 5**
Starting from the equilibrium allocation \((K_E, X_E, \alpha_h^E, \alpha_l^E)\) it is possible to increase \( \alpha_l^E \) by \( \Delta \alpha_l > 0 \) and to adjust \( \alpha_h^E \) by \( \Delta \alpha_h \) in such a way that, after investors revise their positions, and capital prices adjust, social welfare is increased.

\[^{18}\text{We will provide conditions for } \Delta \alpha_h < 0.\]
The proof of Proposition 5 is given in the appendix.

The main intuition for the result is as follows. When banks reallocate capital they exert a negative pecuniary externality on other banks as capital prices decline in downturns and raise in boom periods. Bankers aim at maximizing the expected scale of banks in order to obtain the maximal expected rents. When a negative shock occurs, informational rents are particularly costly for investors. Hence, capital reallocation is comparatively more attractive to motivate investors to finance a larger expected size of the bank. As a consequence, banks sell capital in downturns excessively. The opposite – but to a lesser degree – occurs in boom periods. The associated negative pecuniary externalities on other banks translates into welfare losses. As a consequence, by increasing $\alpha_l$ and adjusting $\alpha_h$, pecuniary externalities are reduced when banks adjust their balance sheets. As downward and upward adjustments are limited, capital prices will fall less in downturns and will increase less in upturns which limits the pecuniary externalities. This is welfare improving. We will provide further intuition in Section 7.3 where we look at the example with $c = 0$.

7.2 One-sided Intervention

We next investigate whether welfare gains can be achieved by increasing $\alpha_l$ alone. In particular, we show in the next proposition that, starting from the competitive equilibrium (characterized in Proposition 4), it is possible to increase the expected social surplus $U$ by increasing $\alpha_l$, i.e. “forcing” banks to invest more during recessions. As we explain later, a similar improvement of social welfare may also be obtained by decreasing $\alpha_h$, i.e. restricting bank lending during booms.

**Proposition 6**

*Starting from the equilibrium allocation $(K^E, X^E, \alpha_h^E, \alpha_l^E)$ it is possible to increase $\alpha_l^E$ by $\Delta \alpha_l > 0$ in such a way that, after investors revise their positions, and capital prices adjust, social welfare is increased.*

The proof is given in the appendix. Proposition 6 shows that it is sufficient to limit excessive downsizing in downturns to achieve welfare gains.
7.3 The Case \( c = 0 \)

It is instructive to focus on the case where \( c = 0 \), i.e. when capital reallocation is costless. In this limit case, the initial lending decision \( k \) becomes irrelevant: the only decision variables that matter are \( k_\eta = (1 + \alpha_\eta)k \), \( \eta = h, l \) at the individual bank level and \( K_\eta = (1 + \alpha_\eta)K \) at the aggregate level. In Appendix B we provide a detailed account of this case and we provide numerical examples. Here we summarize the main insights.

**Summary**

(i) Equilibrium prices are

\[
p_\eta^E = (\eta - 1)\overline{R} + 1 \quad \text{(Lemma 2)}.\]

(ii) If the regulator imposes \( K_l = K_l^E + \Delta K_l \) with \( \Delta K_l > 0 \), \( K_h \) declines if moral hazard in banking is not extremely severe (Proposition 8).

(iii) If the regulator imposes \( K_l = K_l^E + \Delta K_l \) with \( \Delta K_l > 0 \), the resulting change in social welfare \( U \) will be proportional to \( \mathbb{E}[\Delta K_\eta] \) and is positive (Proposition 7).

\[
\Delta U = \mathbb{E}[\eta\overline{R}\Delta K_\eta - p_\eta(\Delta C_0 + \Delta K_\eta)] + \Delta C_0 = (\overline{R} - 1)\mathbb{E}[\Delta K_\eta] > 0 \quad (33)
\]

Moreover, \( \Delta p_l > 0 \) and \( \Delta p_h < 0 \) (Lemma 3).

(iv) The volatility of capital prices in three different scenarios can be expressed as follows with \( \sigma_\eta = \sqrt{q(1-q)(h-l)} \) for the volatility of the macroeconomic shock.

- **First-best allocation**
  \[
  \sigma_p^{FB} = \sqrt{q(1-q)(p_h - p_l)} = \overline{R}\sigma_\eta
  \]

- **Second-best allocation**\(^\text{19} \)
  \[
  \sigma_p^{SB} = \frac{\overline{R}\sigma_\eta}{1 + \frac{\overline{R}-1}{\epsilon r+1}} < \sigma_p^{FB}
  \]
  with \( \epsilon \) being the price elasticity of the supply of capital to the BS.

- **Competitive equilibrium**
  \[
  \sigma_p^{CE} = \overline{R}\sigma_\eta
  \]

\(^{19}\)The second-best allocation is the social planner’s solution subject to the incentive constraint of the banker and the participation constraint of the investor.
(v) The coefficient of variation in the competitive equilibrium is larger than in the first-best and second-best solution.

The summary of the results for the case $c = 0$ confirms and illustrates further the previous results. Moreover, we observe that $\Delta K_h < 0$ if the regulator reduces excessive downsizing and moral hazard is sufficiently severe. Finally, the volatility in the competitive equilibrium is larger than in the second-best solution. Although $\sigma_p^{CE}$ is the same as in the first-best solution, the coefficient of variation, $\frac{\sigma_p}{\bar{p}}$, is highest in the competitive equilibrium, indicating the excess volatility of capital prices in the competitive equilibrium.

In Appendix B we provide an example with the production function $F(X) = A \ln(X + 1)$ with $1 < A < 2$ which allows explicit solutions for $K_h^E, K_l^E, X^E$ and for the equilibrium values when the regulator intervenes by $K_R^E = K_l^E + \Delta K_l$ with $\Delta K_l > 0$. This example illustrates how market participants, prices and allocation react when the regulator reduces excessive downsizing in downturns.

7.4 Macroprudential Regulation

Now the question is: how to force banks to lend more (or rather to reduce lending by a lesser amount) in case of a recession? The answer is a corollary of the implementation of the optimal contract that we proposed in Section 5.3: when investors cannot directly control banks’ investments decisions, they have to rely on short term debt as a way to force bankers to downsize in the bad state. Since social surplus is improved by increasing $\alpha_l$, imposing a regulatory lower bound on short term debt (i.e. a liquidity ratio in the spirit of the Net Stable Funding ratio of Basel 3) would do the job.

An alternative, indirect, way for the regulator to increase social welfare would be to impose a binding capital requirement in state $h$, i.e. contingent a boom. It is easy to see that it would lead to the same effects than the restriction on short term finance discussed above. Note that in our model there is no need for microprudential regulation, since banks are financed by sophisticated investors who can impose a limit to bank leverage. If we introduced as well retail deposits, and thus a rationale for micro-prudential regulation, Proposition 4 could be viewed as establishing the need for a macroprudential regulation in the form of an additional capital requirements only enforced in case of a boom. This is again in the spirit of the Basel 3 proposals.
8 Ramifications and Conclusion

Banks play a dual role in allocating capital in the economy. They lend and reallocate capital across sectors if new information about the productivity of capital arrives. Our main insight is that banks reallocate capital excessively. As a consequence, fluctuations of credit, output and capital prices are excessive. This justifies macroprudential regulation. Numerous further implications and extensions could be pursued. We provide several examples.

Implications

First, as investors hold claims on capital in the FS, the excess volatility of capital prices translates into excess volatility of asset prices. Therefore, our paper provides an explanation why the volatility of asset prices is higher than the volatility of sectoral productivity shocks would suggest. Second, one of our results states that investors will force banks to strongly downsize when negative shocks occur. If banks were buying – voluntarily or forced by regulation – financial securities that provide fresh resources in bad times at the expense of payments in the good state, investors would increase short-term debt to ensure that the same amount of downsizing takes place. Hence, our model implies that banks with insurance against negative macroeconomic shocks have even more short-term debt on their balance sheets.

We have focused on the case where bank capital is scarce. When bank capital would be less scarce, investors may find it attractive to invest all resources into the BS and the FS and defer consumption to the end. In such a setup with positive interest rates, macroprudential regulation of the sort described in the last section continues to be welfare improving if moral hazard still binds.\textsuperscript{20} In addition, macroprudential regulation will have additional repercussions as it affects interest rates. For instance, if additional capital requirements are imposed in a boom, the interest rate is likely to rise, which may be important information for monetary policy makers.

Dynamic Extensions

The model and its mechanism could be embedded in a dynamic macroeconomic model in which payments to bankers constitute the inside equity for the next period. Some bankers may retire, but the others will try to obtain new funding from investors who ponder about reinvesting part of their wealth. Such an extension would create an intertemporal linkage and volatility cycles

\textsuperscript{20}The analysis however, becomes much more cumbersome.
of bank credit. The intuitive reason is as follows. Suppose that in a particular period a negative macroeconomic shock causes excessive downsizing and thus excessively low returns on inside equity. As a consequence (see Corollary 2), with low levels of inside equity in the next period, the volatility of bank credit increases further and downsizing upon negative macroeconomic shocks becomes even more pronounced. Proceeding with this kind of logic suggests the existence of volatility cycles of bank lending. In such circumstances, macroprudential regulation in a particular period does not only avoid excessive volatility of bank lending in this period, but it also reduces the volatility in future periods. A thorough investigation of these intertemporal linkages is left to future research.
Appendix A

Proof of Proposition 5:
We change \( \alpha_t^E \) by some small \( \Delta \alpha_t > 0 \) and we choose an adjustment \( \Delta \alpha_h \) such that investors will continue to provide \( K_t^E \) at \( t = 0 \) to banks. The intervention \( \Delta \alpha_t \) and \( \Delta \alpha_h \) will impact on the prices of capital \( p_\eta \) and the investment \( X \) at date 0 in the frictionless technology. This is because in interior solutions \( (C_0 > 0, K > 0, X > 0) \) competition for funds at \( t = 0 \) imposes that the expected returns on investment in the FS and the BS are equal to 1:

\[
\mathbb{E}[p_\eta] \equiv 1
\]  
(34)

We choose the changes \( \Delta \alpha_t > 0 \) and \( \Delta \alpha_h \) such that the investors continue to offer \( K_t^E \) at \( t = 0 \). Hence,

\[
\mathbb{E}[(1 + \alpha_\eta)(\eta R - b) - \frac{c}{2} \alpha_\eta^2 - p_\eta \alpha_\eta] \equiv 1 - \frac{E}{K_t^E}.
\]  
(35)

As \( \alpha_t \) and \( \alpha_h \) are determined by a social planner, and \( K_t^E \) just fulfills the participation constraint of the investors, the bankers have no possibility to increase their rents. Now:

\[
p_\eta = F'(X - \alpha_\eta K_t^E) \Rightarrow \mathbb{E}[\Delta p_\eta] = \mathbb{E}[F''(X - \alpha_\eta K_t^E)(\Delta X - K_t^E \Delta \alpha_\eta)] = 0
\]  
(36)

Thus

\[
\Delta X = K_t^E \frac{\mathbb{E}[F''(X - \alpha_\eta K_t^E)\Delta \alpha_\eta]}{\mathbb{E}[F''(X - \alpha_\eta K_t^E)]}
\]  
(37)

Totally differentiating condition (35), we get:

\[
\mathbb{E}[(\eta R - b - c \alpha_\eta - p_\eta)\Delta \alpha_\eta - \alpha_\eta \Delta p_\eta] = 0,
\]  
(38)

where

\[
\Delta p_\eta = K_t^E F''(X - \alpha_\eta K_t^E) \left[ \frac{\mathbb{E}[F''(X - \alpha_\eta K_t^E)\Delta \alpha_\eta]}{\mathbb{E}[F''(X - \alpha_\eta K_t^E)]} - \Delta \alpha_\eta \right].
\]  
(39)

Using the equilibrium condition (16), we see that

\[
\eta R - b - c \alpha_\eta^E - p_\eta^E \equiv -\frac{b}{\nu} < 0.
\]  
(40)

Thus the previous condition (38) simplifies to:

\[
\frac{b}{\nu} \mathbb{E}[\Delta \alpha_\eta] = -\mathbb{E}[\alpha_\eta^E \Delta p_\eta].
\]  
(41)
The impact of social welfare is

\[ \Delta U = K^E \mathbb{E}[\{\eta R - c\alpha_\eta - F'(X - \alpha_\eta K^E)\} \Delta \alpha_\eta] + \mathbb{E}[F'(X - \alpha_\eta K^E) - 1] \Delta X. \] (42)

The last term is equal to zero, while the first term can be simplified by inserting the equilibrium values of \( \alpha_\eta \):

\[ \Delta U = K^E b \left( 1 - \frac{1}{\nu} \right) \mathbb{E}[\Delta \alpha_\eta] = (1 - \nu) K^E \mathbb{E}[\alpha_\eta^E \Delta p_\eta] \] (43)

Since \( \nu > 1 \), we just have to check that \( \mathbb{E}[\alpha_\eta^E \Delta p_\eta] < 0 \). Now recall that \( \Delta \alpha_l > 0 \).

**Step 1:**
We first show that \( \Delta p_l > 0 \).

Suppose to the contrary that \( \Delta p_l < 0 \). This would imply that

\[ \Delta X - K^E \Delta \alpha_l > 0 \]

and thus

\[ \Delta X > K^E \Delta \alpha_l > 0 \]

Since \( \Delta p_h > 0 \), as \( \mathbb{E}[\Delta p_\eta] = 0 \), and thus \( \Delta X - K^E \Delta \alpha_h < 0 \), \( \Delta \alpha_h \) has to be positive.

Hence we obtain \( \mathbb{E}[\Delta \alpha_\eta] > 0 \) and \( \mathbb{E}[\alpha_\eta^E \Delta p_\eta] = q \alpha_h^E \Delta p_h + (1 - q) \alpha_l^E \Delta p_l > 0 \) which is a contradiction to the condition (38).

We conclude that \( \Delta p_l > 0 \).

**Step 2:**
From \( \Delta p_l > 0 \) and thus \( \Delta p_h < 0 \) we obtain

\[ \mathbb{E}[\alpha_\eta^E \Delta p_\eta] < 0 \]

and thus the proposition follows.
Proof of Proposition 6:
Increasing $\alpha^{E}$ by $\Delta \alpha_l > 0$ will impact on bankers’ choice of $\alpha^{R}$, the capital prices $p_{\eta}$, investment $X$ at date 0 in the FS and investment $K$ in the BS.

Step 1:
Given a predetermined value $\alpha^{R} = \alpha^{E} + \Delta \alpha_l$, with $\Delta \alpha_l > 0$ and small, the bankers’ problem written at the aggregate level is,

$$\max_{\{\alpha_h, K\}} Kb \left\{ q(1 + \alpha_h) + (1 - q)(1 + \alpha^{R}) \right\}$$

subject to

$$K \left[ q \left\{ (1 + \alpha_h)(\eta R - b) - \frac{c}{2} \alpha^2_h - p_{\eta} \alpha_h \right\} + (1 - q) \left\{ (1 + \alpha^{R})(l R - b) - \frac{c}{2} (\alpha^{R})^2 - p_{\eta} \alpha^{R} \right\} \right] = K - E$$

Denoting by $L$ the associated Lagrangian, and by $\lambda$ the multiplier, we obtain the first order conditions:

$$\frac{\partial L}{\partial K} = bE[1 + \alpha_{\eta}] + \lambda E[(1 + \alpha_{\eta})(\eta R - b) - \frac{c}{2} \alpha^2_{\eta} - p_{\eta} \alpha_{\eta}] - \lambda = 0 \quad (44)$$

$$\frac{\partial L}{\partial \alpha_h} = qK [b + \lambda (h R - b - c \alpha_h - p_h)] = 0, \quad (45)$$

From the second condition we obtain,

$$p_{h} = h R - b - c \alpha_h + \frac{b}{\lambda}$$

Inserting this expression and the equilibrium condition

$$p_{l} = \frac{1 - q p_{h}}{1 - q}$$

into the first condition yields

$$1 - \frac{b}{\lambda} (1 + \overline{\pi}) = E[(\eta R - b)(1 + \alpha_{\eta}) - \frac{c}{2} \alpha^2_{\eta}] - \alpha^{R} - q(\alpha_h - \alpha^{R})(h R - b - c \alpha_h + \frac{b}{\lambda})$$

After rearranging terms we obtain,

$$\frac{b}{\lambda} \left\{ (1 + q \alpha_h + (1 - q) \alpha^{R} - q \alpha_h + q \alpha^{R}) = 1 + \alpha^{R} + b \left[ 1 + q \alpha_h + (1 - q) \alpha^{R} - q \alpha_h + q \alpha^{R} \right] \right.$$  

$$- \overline{R} \left[ q h + q h \alpha_h + (1 - q) l + (1 - q) l \alpha^{R} - q h \alpha_h + q h \alpha^{R} \right]$$  

$$+ \frac{c}{2} \left[ q \alpha^2_h + (1 - q)(\alpha^{R})^2 - 2 q \alpha^2_h + 2 q \alpha^{R} \alpha_h \right]$$

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After simplification we obtain,
\[ \frac{b}{\lambda} = 1 + b - \bar{R} + \frac{c}{2} \left[ \frac{(\alpha^R_i)^2 - q(\alpha_h - \alpha^R_i)^2}{1 + \alpha^R_i} \right] \]

We denote,
\[ \psi(\alpha^R_i, \alpha_h) := \frac{(\alpha^R_i)^2 - q(\alpha_h - \alpha^R_i)^2}{1 + \alpha^R_i} \]

Hence, we obtain
\[ p_h = (h - 1)\bar{R} + 1 + \frac{c}{2} \left( \psi(\alpha^R_i, \alpha_h) - 2\alpha_h \right) \quad (46) \]

For given \((p_h, \alpha^R_i)\), equation (46) uniquely determines \(\alpha_h\). The solution is denoted by \(\alpha_h(p_h, \alpha^R_i)\).

**Step 2:**
For every choice \(\alpha^R_i\) we can define an equilibrium as an array \(\Sigma = \{C_0, K, X, \alpha_h, p_h, p_l | \alpha^R_i\}\) such that
(i) \(K = \frac{E}{\rho(p|\alpha^R_i)}\)
where \(\rho(p|\alpha^R_i) = 1 - q \left\{ (1 + \alpha_h(p_h, \alpha^R_i))(h\bar{R} - b) - \frac{c}{2} \alpha^2_h(p_h, \alpha^R_i) - p_h \alpha_h(p_h, \alpha^R_i) \right\} \)
\[ - (1 - q) \left\{ (1 + \alpha^R_i)(l\bar{R} - b) - \frac{c}{2} (\alpha^R_i)^2 - p_l \alpha^R_i \right\} \]
(ii) Investments in the BS are adjusted in \(t = 1\) by \(\alpha_h(p_h, \alpha^R_i), \alpha^R_i\).

(iii) \(p_h = F'(X - \alpha_h(p_h, \alpha^R_i)K)\)
\(p_l = F'(X - \alpha^R_iK)\)
(iv) \(p_l = \frac{1 - q p_h}{1 - q}\)
(v) \(K + X + C_0 = 1\)
(vi) \(C_0 > 0, K > 0, X > 0\).

**Step 3:**
For \(\alpha^R_i = \alpha^E_i\) the equilibrium defined in Step 2 coincides with the equilibrium without intervention.

**Step 4:**
We next show that for \(\alpha^R_i = \alpha^E_i + \Delta \alpha_i\) we have \(\Delta p_l > 0, \Delta p_h < 0\).

Suppose that this is not true and \(\Delta p_l < 0, \Delta p_h > 0\). Then, the following lemma holds.
Lemma 1
\( \Delta K < 0. \)

The proof of the lemma will be given at the end of this proof. With Lemma 1 we obtain,
\[
\Delta X - \Delta \alpha_l K - \alpha_l^E \Delta K > 0
\]
\[
\Delta X - \Delta \alpha_l K > 0
\]
From
\[
\Delta X < \Delta \alpha_h K + \alpha_h^E \Delta K
\]
we obtain
\[
\Delta X < \Delta \alpha_h K
\]
Hence, \( \Delta \alpha_h > \Delta \alpha_l > 0. \) Then, from (46) we obtain
\[
\Delta p_h = -c\Delta \alpha_h + \frac{c}{2} \left[ \psi(\alpha_l^E + \Delta \alpha_l, \alpha_h^E + \Delta \alpha_h) - \psi(\alpha_l^E, \alpha_h^E) \right].
\]
As \( \Delta \alpha_h > \Delta \alpha_l > 0 \) we observe that
\[
\psi(\alpha_l^E + \Delta \alpha_l, \alpha_h^E + \Delta \alpha_h) < \psi(\alpha_l^E, \alpha_h^E).
\]
Hence, \( \Delta p_h < 0, \) which is a contradiction. We conclude that \( \Delta p_l > 0, \Delta p_h < 0. \)

Step 5:
With \( \Delta p_l > 0, \Delta p_h < 0, \) and thus less volatility of capital prices and more favorable prices for reallocation decisions of banks, similar arguments as in Proposition 5 establish that social welfare will increase.

Proof of Lemma 1:
Suppose \( \Delta p_l < 0 \) and \( \Delta p_h > 0. \) This implies
\[
\Delta X - K \Delta \alpha_l - \Delta K \alpha_l^E > 0 \quad (47)
\]
\[
\Delta X - K \Delta \alpha_l - \Delta K \alpha_h^E < 0 \quad (48)
\]
As prices for bankers are less favorable in the bad state and the good state, his expected payoff has to decline. Otherwise his choices \((K^E, \alpha^E_h, \alpha^E_l)\) could not be optimal. Hence, 

\[(K^E + \Delta K)(q(1 + \alpha^E_h + \Delta \alpha_h) + (1 - q)(1 + \alpha^E_l + \Delta \alpha_l)) < K^E(q(1 + \alpha^E_h) + (1 - q)(1 + \alpha^E_l))\]

which yields

\[\Delta K(1 + \alpha^E_l) + (K^E + \Delta K)(q\Delta \alpha_h + (1 - q)\Delta \alpha_l) < 0\]

As \(\Delta \alpha_l, \Delta K\) and \(\Delta \alpha_h\) are small, we neglect all terms with \(\Delta K \Delta \alpha_l\) and \(\Delta K \Delta \alpha_h\). From (47) and (48) we obtain

\[K^E \Delta \alpha_l + \Delta K \alpha^E_l < K^E \Delta \alpha_h + \Delta K \alpha^E_h\]

or

\[K^E \Delta \alpha_h > K^E \Delta \alpha_l + \Delta K(\alpha^E_l - \alpha^E_h)\]

Hence, it must hold

\[\Delta K(1 + \alpha^E_l) + \Delta K \alpha^E_l < 0\]

or

\[\Delta K(1 + q\alpha^E_h + (1 - q)\alpha^E_l + q\alpha^E_l - q\alpha^E_h) + \Delta \alpha_l K^E < 0\]

which yields,

\[\Delta K(1 + \alpha^E_l) + \Delta \alpha_l K^E < 0.\]

As \(\alpha^E_l > -1\) and \(\Delta \alpha_l > 0\) we obtain

\[\Delta K < 0.\]
Appendix B - Zero Adjustment Costs

B1) The Problem

With \( c = 0 \), it does not matter in which sectors investments in \( t = 0 \) take place. Suppose for ease of presentation that no resources are invested into the BS at \( t = 0 \) and that the amount of capital \( k_h \) and \( k_l \), respectively are bought by bankers from the FS in \( t = 1 \). Then the contractual problem between each banker and its financiers can be formulated as follows.

\[
\max_{k_h, k_l} [qk_h + (1 - q)k_l]b
\]

s.t. \( qk_h(h \bar{R} - b) + (1 - q)k_l(l \bar{R} - b) = qp_hk_h + (1 - q)p_lk_l - E \) \hspace{1cm} (49)

The constraint says that the expected return to investors has to be equal to the amount of capital that has to be bought, minus the wealth provided by bankers.

The constraint yields,

\[
(1 - q)k_l[l \bar{R} - b - p_l] = qk_h[p_h + b - h \bar{R}] - E \hspace{1cm} (50)
\]

Hence, the problem can be rewritten as

\[
\max_{k_h} \left[ qk_h \frac{p_h - p_l + l \bar{R} - Rh}{l \bar{R} - b - p_l} - \frac{E}{l \bar{R} - b - p_l} \right] b
\]

We denote by \( K_h \) and \( K_l \) the aggregate investments by banks upon the realization of the macroeconomic shock. We obtain

B2) Equilibria

Lemma 2

In any equilibrium we have

(i) \( h \bar{R} - p_h = l \bar{R} - p_l \)

(ii) \( p_h = (h - 1) \bar{R} + 1 = F'(X - K_h) \)

\( p_l = (l - 1) \bar{R} + 1 = F'(X - K_l) \)

Proof of Lemma 2:

The first point follows from the equilibrium requirement that \( K_h \) has to be positive and finite.
If (i) does not hold, bankers would either choose $K_h = 0$ or $K_h = \infty$. For the second point we observe that the equilibrium condition in the first part of the lemma and condition (50) imply

$$(h\bar{R} - p_h)(1 - q) = l\bar{R}(1 - q) - 1 + qp_h$$

With $qh + (1 - q)l = 1$ we obtain

$$p_h = (h - 1)\bar{R} + 1,$$

and $E[p_q] = 1$ implies,

$$p_l = (l - 1)\bar{R} + 1,$$

We next observe that condition (ii) of Lemma 2 uniquely determine $X - K_h$ and $X - K_l$. Together with condition (50) we obtain the equilibrium value of $X$ and thus the equilibrium is unique. The equilibrium values are denoted by $X^E, K^E_h, K^E_l, p^E_h$ and $p^E_l$.

**B3) Intervention**

We next study intervention and assume that $K_l$ is increased by $\Delta K_l$ ($\Delta K_l > 0$), causing changes in $\Delta K_h, \Delta p_h, \Delta p_l$ and $\Delta X$. The participation constraint of investors becomes

$$E = q(K_h + \Delta K_h)(p_h + \Delta p_h + b - h\bar{R}) + (1 - q)(K_l + \Delta K_l)(p_l + \Delta p_l + b - l\bar{R})$$  \hspace{1cm} (51)

The remaining equilibrium conditions are

$$q\Delta p_h + (1 - q)\Delta p_l = 0$$  \hspace{1cm} (52)

$$p_h + \Delta p_h = F'(X + \Delta X - K_h - \Delta K_h)$$  \hspace{1cm} (53)

$$p_l + \Delta p_l = F'(X + \Delta X - K_l - \Delta K_l)$$  \hspace{1cm} (54)

We observe that prices for capital increase in the bad state.

**Lemma 3**

$\Delta p_l > 0, \Delta p_h < 0$
Proof of Lemma 3:
Suppose to the contrary that $\Delta p_l < 0$. This would imply

$\Delta X - \Delta K_l > 0$

As $\Delta p_h > 0$ would have to hold, $\Delta X - \Delta K_h < 0$. As $\Delta X > \Delta K_l > 0$, $\Delta K_h > 0$.

We rewrite condition (51) as follows, where we neglect all terms of the type $\Delta K_\eta \Delta p_\eta$

$q \left[ \Delta K_h(p_h + b - hR) + \Delta p_hK_h \right] + (1 - q) \left[ \Delta K_l(p_l + b - lR) + \Delta p_lK_l \right] = 0 \quad (55)$

Using $\Delta K_l > 0$, $\Delta K_h > 0$ and $p_h + b - hR > 0$, $p_l + b - lR > 0$ we must have

$q\Delta p_hK_h + (1 - q)\Delta p_lK_l < 0$

as we have neglected positive terms on the left hand side. As $K_h > K_l$ from Lemma 2 (ii) and $\mathbb{E}[\Delta p_\eta] = 0$ we obtain a contradiction as $\Delta p_h > 0$ and $\Delta p_l < 0$.

Hence we conclude $\Delta p_l > 0$ and $\Delta p_h < 0$.

\[\square\]

Proposition 7
$\Delta U > 0$.

Proof of Proposition 7:
We show that $\mathbb{E}[\Delta K_\eta] > 0$ which establishes $\Delta U > 0$. The participation constraint of the investor yields

$q\Delta K_h(p_h + b - hR) + qK_h\Delta p_h + (1 - q)\Delta K_l(p_l + b - lR) + (1 - q)K_l\Delta p_l = 0$

Using the equilibrium values for $p_h$ and $p_l$ yields

$(b + 1 - lR)\mathbb{E}[\Delta K_\eta] + qK_h\Delta p_h + (1 - q)K_l\Delta p_l = 0$

As $q\Delta p_h + (1 - q)\Delta p_l = 0$, $K_h > K_l$, $\Delta p_h < 0$, $\Delta p_l > 0$ we obtain $\mathbb{E}[\Delta K_\eta] > 0$ as $b + 1 - lR > 0$.

\[\square\]

Finally, we obtain
Proposition 8

Suppose that $1 + b < \bar{R} [1 + \beta (1 - l)]$. Then, $\Delta K_h < 0$ where,

$$\beta = \frac{F''_h F''_l}{E[F''_\eta F''_\Theta]}, \quad F''_\eta = F'' (X - K_\eta)$$

and $\Theta$ is defined by,

$$K_h - K_l = (h - l) S'(\Theta).$$

Proposition 8 shows that $\Delta K_h < 0$ if the moral hazard problem is not extremely severe.

Proof of Proposition 8:

We consider the participation constraint of the investors

$$E = E [(b + p_\eta - \eta \bar{R}) K_\eta]$$

The constraint will be binding after the intervention. Hence, using Lemma 2 for the prices $p_\eta$ yields

$$E[\Delta K_\eta (b + 1 - \bar{R})] + E[\Delta p_\eta K_\eta] = 0 \quad (56)$$

We next calculate the price changes. Using the formulas in the proof of Proposition 5 we obtain

$$\Delta X = \frac{E[F''_\eta \Delta K_\eta]}{E[F''_\eta]}$$

where $F''_\eta = F''(X - K_\eta)$. Now,

$$\Delta p_\eta = (\Delta X - \Delta K_\eta) F''_\eta$$

Hence,

$$\Delta p_h = \frac{1}{E[F''_\eta]} \left\{ q F''_h F''_h \Delta K_h + (1 - q) F''_h F''_l \Delta K_l - q F''_h F''_h \Delta K_h - (1 - q) F''_h F''_l \Delta K_h \right\}$$

$$= \frac{(1 - q) F''_h F''_l}{E[F''_\eta]} (\Delta K_l - \Delta K_h)$$

Similarly,

$$\Delta p_l = \frac{q F''_h F''_l}{E[F''_\eta]} (\Delta K_h - \Delta K_l)$$

Inserting the price changes into (56) yields,

$$q \Delta K_h \left[ 1 + b - \bar{R} - \frac{(1 - q) F''_h F''_l}{E[F''_\eta]} (K_h - K_l) \right] + (1 - q) \Delta K_l \left[ 1 + b - \bar{R} + \frac{q F''_h F''_l}{E[F''_\eta]} (K_h - K_l) \right] = 0$$

39
Since $K_h > K_l$, $\Delta K_l > 0$, a sufficient condition for $\Delta K_h > 0$ is,

$$Z_h := 1 + b - \overline{R} + \frac{qF''_hF''_l}{E[F''_{\eta}]}(K_h - K_l) < 0$$

Lemma 2, equation (29) and the Taylor expansion yields

$$K_h - K_l = S(p_h) - S(p_l) = S((h-1)\overline{R} + 1) - S((l-1)\overline{R} + 1) = (h - l)\overline{R}S'(\Theta)$$

for some $\Theta \in [p_l, p_h]$. Hence,

$$Z_h = 1 + b - \overline{R} - q(h - l)\overline{R} \frac{F''_hF''_l}{E[F''_{\eta}]}$$

where we have used that $S(\Theta) = 1 - F^{\ell-1}(\Theta)$ and thus, $S'(\Theta) = -\frac{1}{F''(\Theta)}$.

We define,

$$\beta := \frac{F''_hF''_l}{E[F''_{\eta}]}$$

and obtain $Z_h < 0$ if and only if

$$\beta > \frac{1 + b - \overline{R}}{q(h - l)\overline{R}}$$

or,

$$1 + b < \overline{R}[1 + \beta(1 - l)]$$

which establishes the proposition.

B4) Volatility

We calculate the volatility of capital prices in three different versions of the model. In the competitive equilibrium from Lemma 2 we obtain for the volatility of capital prices, denoted by $\sigma_p^{CE}$,

$$\sigma_p^{CE} = \sqrt{q(1-q)(p_h - p_l)} = \overline{R}\sigma_\eta$$

(57)

where $\sigma_\eta = \sqrt{q(1-q)(h - l)}$ is the volatility of the macroeconomic shock.
We next calculate the volatility in the first-best solution. The first-best solution is characterized by

\[
C_0 = 0 \\
p_\eta = \eta R = F'(1 - K_\eta) \\
K_\eta = S(\eta R).
\]

Hence, volatility of capital prices is given by

\[
\sigma_{p}^{FB} = R\sigma_\eta.
\]

Finally, we look at the second-best solution in which the social planner can only control aggregate investments in the BS. The second-best solution is the solution of the following:

\[
\max_{K_h,K_l} U = C_0 + E[\eta RK_\eta + F(1 - C_0 - K_\eta)] \\
\text{s.t. } E = E[K_\eta(F'(1 - C_0 - K_\eta) + b - \eta R)].
\]

We denote by \( \hat{\lambda} \) the Lagrange multiplier. The first-order conditions amount to

\[
\eta R - p_\eta - \hat{\lambda}[p_\eta + b - \eta R - K_\eta F''_\eta] = 0
\]

where \( p_\eta = F'(1 - C_0 - K_\eta) \) is the price of capital and \( F''_\eta \) is the abbreviation of \( F''(1 - C_0 - K_\eta) \).

Together with the equilibrium condition that investors have to be indifferent between consuming and investing in the FS, i.e. \( E[p_\eta] = 1 \), we obtain for the price in the second-best solution

\[
p_\eta^{SB} = \eta R - \frac{\hat{\lambda}[b - K_\eta F''_\eta]}{1 + \hat{\lambda}}.
\]  

(58)

As \( E[p_\eta^{SB}] = 1 \), we obtain

\[
1 = \frac{\hat{\lambda}[b - E[K_\eta F''_\eta]]}{1 + \hat{\lambda}}
\]  

(59)

By

\[
p_\eta^{SB} = 1 + (\eta - 1)R + \frac{\hat{\lambda}}{1 + \hat{\lambda}}[K_\eta F''_\eta - E[K_\eta F''_\eta]]
\]

\[
\frac{\hat{\lambda}}{1 + \hat{\lambda}} = \frac{R - 1}{b - E[K_\eta F''_\eta]}
\]
and

\[ p^S_B = 1 + (\eta - 1)\bar{R} + \frac{\bar{R} - 1}{b - \mathbb{E}[K^F_{\eta}]^{-1}} [K^F_{\eta} - \mathbb{E}[K^F_{\eta}]] \]

we introduce \( \varepsilon_{\eta} = \frac{p_{n}}{S(p_{n})} \frac{dS}{dp_{n}} = -\frac{F'_{\eta}}{F''_{\eta}} \) for the price elasticity of the supply of capital to the BS. Using \( K^F_{\eta} = -\frac{p^S_B}{\varepsilon_{\eta}} \), the price formula can be rewritten as

\[ p^S_B = 1 + \frac{(\eta - 1)\bar{R}}{b + \mathbb{E} \left[ \frac{p^S_B}{\varepsilon_{\eta}} \right]} \left[ p^S_B - \mathbb{E} \left[ \frac{p^S_B}{\varepsilon_{\eta}} \right] \right] \]  

(60)

If we assume that the price elasticity \( \varepsilon_{\eta} \) is independent of the macroeconomic shock, we can simplify the formula. Let \( \varepsilon \equiv \varepsilon_{\eta}, \eta \in \{l, h\} \), then, using \( \mathbb{E}[p^S_B] = 1 \) yields

\[ p^S_B = 1 + (\eta - 1)\bar{R} - \frac{\bar{R} - 1}{b\varepsilon + 1} [p^S_B - 1] \]

and

\[ p^S_B = \frac{1 + (\eta - 1)\bar{R}}{1 + \frac{\bar{R} - 1}{b\varepsilon + 1}} + \frac{\bar{R} - 1}{b\varepsilon + 1 + \bar{R} - 1} \]

Finally the volatility is given by

\[ \sigma^S_B = \frac{\bar{R}\sigma_{\eta}}{1 + \frac{\bar{R} - 1}{b\varepsilon + 1}} \]

which is smaller than \( \sigma_{F B}^s \) and \( \sigma_{C E}^s \).

**B5) An Example**

As an example we choose \( F(X) = A \ln(X + 1) \) with \( 1 < A < 2 \).

**Equilibria**

From Lemma 2, we observe that prices are independent of the particular production function in the FS. From Lemma 2 (ii) we obtain

\[ K_h = X + 1 - \frac{A}{(h - 1)\bar{R} + 1} \]

\[ K_l = X + 1 - \frac{A}{(l - 1)\bar{R} + 1} \]
Using the participation constraint (50) at the aggregate level, we obtain

\[ E = (1 + b - \bar{R})(qK_h + (1 - q)K_l) \]
\[ = (X + 1)(1 + b - \bar{R}) - A(1 + b - \bar{R}) \left( \frac{q}{(h - 1)\bar{R} + 1} + \frac{1 - q}{(l - 1)\bar{R} + 1} \right) \]
\[ = (1 + b - \bar{R}) \left( X + 1 - A \left[ \frac{q}{(h - 1)\bar{R} + 1} + \frac{1 - q}{(l - 1)\bar{R} + 1} \right] \right) \]

Hence,

\[ X^E = \frac{E}{1 + b - \bar{R}} + A \left( \frac{q}{(h - 1)\bar{R} + 1} + \frac{1 - q}{(l - 1)\bar{R} + 1} \right) - 1 \]  
(61)

\[ K^E_h = \frac{E}{1 + b - \bar{R}} + \frac{A(1 - q)(h - l)\bar{R}}{[(l - 1)\bar{R} + 1][(h - 1)\bar{R} + 1]} \]  
(62)

\[ K^E_l = \frac{E}{1 + b - \bar{R}} + \frac{Aq(l - h)\bar{R}}{[(l - 1)\bar{R} + 1][(h - 1)\bar{R} + 1]} \]  
(63)

We observe that

\[ \frac{\partial X^E}{\partial b} = \frac{\partial K^E_h}{\partial b} = \frac{\partial K^E_l}{\partial b} = \frac{-E}{(1 + b - \bar{R})^2} \]

**Intervention**

Suppose that \( K_l \) is increased by \( \Delta K_l \) with \( \Delta K_l \) being small. Hence, we can neglect all terms \( \Delta p_\eta \Delta K_\eta, \Delta p_\eta \Delta X \). Solving (52) to (54) yields

\[ \Delta p_l = \frac{M \pm \sqrt{M^2 - 4LN}}{2L} \]

where

\[ L = \frac{(1 - q)A[(h - 1)\bar{R} + 1][(l - 1)\bar{R} + 1]}{(1 - \bar{R} + b) \left\{ Aq(h - l)\bar{R} - \Delta K_l[(h - 1)\bar{R} + 1][(l - 1)\bar{R} + 1] \right\} + 1 - q} \]

\[ M = \frac{qA[(h - 1)\bar{R} + 1][(l - 1)\bar{R} + 1]}{Aq(h - l)\bar{R} - \Delta K_l[(h - 1)\bar{R} + 1][(l - 1)\bar{R} + 1]} \left[ 1 + \frac{(1 - q)(h - l)\bar{R}}{(1 - \bar{R} + b)} \right] + q[(h - 1)\bar{R} + 1] - (1 - q)((l - 1)\bar{R} + 1) \]
\[N = q[(h-1)R + 1][(l-1)R + 1]\left[\frac{qA(h-l)R}{Aq(h-l)R - \Delta K_I[(h-1)R + 1][(l-1)R + 1]} - 1\right]\]

\[
\Delta p_h = \frac{(q-1)\Delta p_l}{q} \\
\Delta X = \frac{A}{(l-1)R + 1 + \Delta p_l} + K^E_I + \Delta K_l - 1 - X \]

\[
\Delta K_h = X^E + \Delta X + 1 - K^E_h - \frac{A}{(h-1)R + 1 + \Delta p_h}
\]

where the values of \(X^E, K^E_I\) and \(K^E_h\) are given in equations (61) to (63).
Appendix C - List of Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS</td>
<td>Frictionless sector with a technology that exhibits decreasing marginal returns</td>
</tr>
<tr>
<td>BS</td>
<td>Banking sector that is subject to macroeconomic risk</td>
</tr>
<tr>
<td>$e$</td>
<td>Wealth of an individual banker</td>
</tr>
<tr>
<td>$E$</td>
<td>The aggregate wealth of bankers in $t = 0$</td>
</tr>
<tr>
<td>$U$</td>
<td>The social welfare</td>
</tr>
<tr>
<td>$C_t$</td>
<td>Consumption in period $t$ with $t = 0, 2$</td>
</tr>
<tr>
<td>$k$</td>
<td>The investment of a single bank in the BS in $t = 0$</td>
</tr>
<tr>
<td>$K$</td>
<td>The aggregate amount of investment in the BS in $t = 0$</td>
</tr>
<tr>
<td>$\tilde{R}$</td>
<td>The idiosyncratic return of the the BS</td>
</tr>
<tr>
<td>$\bar{R}$</td>
<td>The expectation of the idiosyncratic return $\tilde{R}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Macroeconomic shock with $\eta = h$ (high) with probability $q$ and $\eta = l$ (low) with probability $1 - q$</td>
</tr>
<tr>
<td>$m$</td>
<td>The expectation of the macroeconomic shock, i.e. $m = qh + (1 - q)l = 1$</td>
</tr>
<tr>
<td>$X$</td>
<td>The aggregate amount of investment in the FS in $t = 0$</td>
</tr>
<tr>
<td>$F(X)$</td>
<td>Output of the FS in $t = 2$, if $X$ is invested in the FS in $t = 0$</td>
</tr>
<tr>
<td>$G(x)$</td>
<td>The distribution function of $x$ where $x$ is the productivity of an agent in the FS</td>
</tr>
<tr>
<td>$p_\eta$</td>
<td>The (interim) rate of return on capital when macroeconomic shock is $\eta$</td>
</tr>
<tr>
<td>$r$</td>
<td>The interest rate for capital with expected rate of return on capital $E[p_\eta] = 1 + r$</td>
</tr>
<tr>
<td>$\alpha_\eta$</td>
<td>The adjustment to the investment in the BS in $t = 1$. $\alpha_\eta &gt; 0$ means additional investment in the BS and $\alpha_\eta &lt; 0$ denotes disinvestment</td>
</tr>
<tr>
<td>$c(\alpha)$</td>
<td>Adjustment cost for reallocating capital across sectors</td>
</tr>
<tr>
<td>$o, S, F$</td>
<td>$o$ denotes the project outcome with $o = S$ indicating success and $o = F$ indicating a failure</td>
</tr>
<tr>
<td>$\tau$</td>
<td>The probability with which $o = S$ occurs if the banker exerts effort</td>
</tr>
<tr>
<td>$\tau - \Delta$</td>
<td>The probability with which $o = S$ occurs if the banker shirks</td>
</tr>
<tr>
<td>$b^o_\eta$</td>
<td>Payment to the banker in $t = 2$ per unit of investment in the BS ($o = S, F$) if $\eta$ occurs</td>
</tr>
<tr>
<td>$C(k, \alpha_\eta, b^o_\eta)$</td>
<td>The financial contract between a banker and investor</td>
</tr>
<tr>
<td>$B$</td>
<td>Private benefits per unit of investment in the BS obtained by the banker by shirking</td>
</tr>
<tr>
<td>$R^o$</td>
<td>The idiosyncratic return $\tilde{R}$ for project outcome $o = S, F$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>The standard deviation of credit growth $\alpha_\eta$</td>
</tr>
<tr>
<td>$\bar{\alpha}$</td>
<td>The expectation of $\alpha_\eta$</td>
</tr>
<tr>
<td>$\alpha_l$</td>
<td>$\alpha_l &gt; -1$ is the lower bound of $\alpha_l$ such that output in the BS is non-negative.</td>
</tr>
</tbody>
</table>
The supply of capital to the BS, as a function of capital prices $p$.

The expected payment to the banker per unit of asset, with $b = \frac{\tau_B}{\Delta}\rho_0(r)$.

The market imposed capital ratio ($\frac{\tau_B}{\Delta}$) in the rigid economy.

The repayment amount of the riskless deposits issued in the rigid economy in $t = 0$.

The repayment amount of the riskless deposits issued in the rigid economy in $t = 1$.

The fraction of the total dividends paid to the banker as inside equity in the rigid economy.

The market imposed capital ratio for privately optimal contracts.

The amount of short term debt (repaid in $t = 1$) issued for privately optimal contracts.

The amount of long term debt (repaid in $t = 2$) issued for privately optimal contracts.

The amount of convertible bonds issued for privately optimal contracts.

where each convertible bond is converted to $y$ shares.

The new debt issued in $t = 1$ for privately optimal contracts if state $h$ occurs.

The total payment to debt holders in $t = 2$ for privately optimal contracts.

where $\eta = h, l$ and $o = S, F$.

The aggregate output in state $\eta$.

The Lagrangian multiplier of the maximization problem in the competitive equilibrium.

The probability that $\eta$ occurs.

A decreasing function of $\sigma$, which gives a unique solution $\sigma^E$, that determines

the unique competitive equilibrium.

The parameter values of the competitive equilibrium, where $I = K, X, \alpha, p, \sigma$.

The change from the equilibrium capital adjustment as a result of intervention.

The change from the equilibrium capital prices as a result of intervention.

The change from the equilibrium capital allocations as a result of intervention.

where $I = K, X, C_0$.

The volatility of the macroeconomic shock $\eta$.

The volatility of capital prices in the first best allocation.

The volatility of capital prices in the second best allocation.

The volatility of capital prices in the competitive equilibrium.

The price elasticity of the supply of capital to the BS.

The capital in the BS after the regulator intervenes with $K_i^R = K_i^E + \Delta K_i$ in the case $c = 0$.

The Lagrangian multiplier of the maximization problem with one sided intervention.

The capital adjustment after one sided intervention with $\alpha_i^R = \alpha_i^E + \Delta \alpha_i$.

A function defined as $\psi(\alpha_i^R, \alpha_h) := \frac{(\alpha_i^R)^2 - q(\alpha_h - \alpha_i^R)^2}{1 + \alpha_i^R}$.

A positive value defined as $\beta = \frac{F''(X)}{E[F''_\eta | F''_\epsilon]}$, where $F''_\eta = F''(X - K_\eta)$.

$\Theta \in [p_h, p_l]$ defined by $K_h - K_i = (h - l)S'(\Theta)$.
\dot{\lambda} \quad \text{The Lagrangian multiplier of the maximization problem of the second best solution}

A \quad \text{A constant with } 1 < A < 2 \text{ used in the functional form of } F(X) \text{ of the example such that } F(X) = A \ln(X + 1)

L, M, N \quad \text{Constants used in the example}
References


